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Comparing seismic hazard software packages: M3C vs. OpenQuake

Earth Hazards and Observatories Programme

Open Report OR/19/038

BRITISH GEOLOGICAL SURVEY

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Comparing seismic hazard software packages: M3C vs. OpenQuake

I Mosca

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OpenQuake is an open source software available (https://www.globalquakemodel.org/oq-get-started - released under https://www.openhub.net/p/oq-engine).

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Summary

This report describes a comparison of the software M3C with the software OpenQuake that have been developed for seismic hazard and risk assessment (Pagani at al., 2014). The comparison is made in terms of methodology, IT functionalities of the software packages and hazard results.

The goal of the report is to show that the code M3C, which was developed at the British Geological Survey (BGS) in the second half of the 1990s for probabilistic seismic hazard assessment, compares well with more modern methods as a result of regular updates to incorporate advances in seismic hazard analysis and other state-of-art techniques. The frequent testing and quality assurance of the new features of the code ensures that it is an excellent tool for assessing seismic hazard for commercial and academic projects.

We perform a comparison between M3C and OpenQuake using the source model of the UK developed by Musson and Sargeant (2007). OpenQuake is an open-source software package for seismic hazard and risk calculations developed by the Global Earthquake Model initiative (Crowley et al., 2013). We perform many tests to compare the implementation of the basic steps of the probabilistic seismic hazard assessment in the two software packages, including the ground motion prediction equations, the fault rupture modelling, and the treatment of epistemic uncertainties in the recurrence statistics.

The main conclusion from the present work is that if input parameters are identical, the outputs from the two software packages are in excellent agreement. When I estimate the relative difference between the outputs, the agreement is good for annual probabilities of exceedance between 10^{-2} and 10^{-5}, i.e. the range of interest of earthquake engineering, in spite of the differences in the implementation of the methodology and the IT functionalities of M3C and OpenQuake. The discrepancies between the results are explained by: 1) the different magnitude scaling relationship adopted by M3C and OpenQuake; and 2) the use of ground motion predictive equations based on the rupture distance, rather than the Joyner-Boore distance.

1 Introduction

In the last thirty years, many studies for assessing probabilistic seismic hazard have been published (for reviews see Reiter, 1990; Abrahamson, 2000; McGuire, 2004; Bommer et al., 2005) where different criteria are used for characterizing the seismic source zone model (defined by source geometry and source parameters, such as maximum magnitude, recurrence statistics and rupture geometry), the selection of the ground motion prediction equations (GMPEs) for the study area, the treatment of (aleatory and epistemic) uncertainty, and the approach to compute probabilistic seismic hazard assessment (PSHA; e.g. Cornell-McGuire PSHA, and Monte Carlo based PSHA). Even if users select the same method for PSHA and use the same criteria for the required input, further discrepancies may arise from computational aspects of the engine used to encode the PSHA method, such as programming language, coding strategies for numerical integrations and numerical tolerance of the computer program.

The first public domain computer code for seismic hazard assessment was EQRISK developed by McGuire (1976), later modified in FRISK by McGuire (1978). Since the second half of the 1970s a large number of software packages and codes have been published, e.g. SeisRisk (Bender and Perkins, 1982), PRISK (Principia Mechanica LTD, 1985), NSHMP (Frankel et al., 2002), OpenSHA (Field et al., 2003), EQRM (Robinson et al., 2006), M3C (Musson, 1999, 2009), CRISIS (Ordaz et al., 2013), EqHaz (Assatourians and Atkinson, 2013) and OpenQuake (Pagani et al., 2014). Consequently, there are also many studies that compare and validate software packages for PSHA (e.g. Danciu et al., 2010; Thomas et al. 2010; Musson, 2012; Bommer et al., 2013; Monelli et al., 2014). For example, Thomas et al. (2010) compare many free and commercial software packages for PSHA using a simple configuration of areal and fault sources. They find that hazard curves calculated by different codes may diverge even for simple source-site configurations due to the numerical approaches used to solve particular mathematical problems, e.g. the presence or lack of a leaky boundary for fault rupture and the lower limit of integration for the hazard (Thomas et al., 2010). Their verification process can be used to validate current and future codes for PSHA. Danciu et al. (2010) present a review of non-commercial computer programs for PSHA in terms of IT functionalities, methodological aspects of PSHA, and benchmarking exercises. The main conclusion from their study is that a software for PSHA must be open-source, flexible (i.e. it is straightforward to implement new input models and new features), user-friendly, verified (i.e. it should be verified against other codes), and should include the basic seismic hazard requirements (e.g. it should include hazard curves, spectra, maps, and disaggregation of the seismic hazard results, incorporate easily new GMPEs, and account for epistemic uncertainties).

The software M3C for the Monte Carlo-based PSHA was developed by the British Geological Survey (BGS) in the second half of the 1990s (e.g. Musson, 1999, 2000). Since then, BGS has routinely undertaken commercial seismic hazard work for engineering, insurance or government projects worldwide using this code (e.g. Musson et al., 2006; Musson & Sargeant, 2007). The goal of the present report is to show that the software M3C is a rigorously tested and state-of-art code that incorporates the recent advances in seismic hazard analysis and therefore its performance is as good as that of recently published software packages.

Musson (2012) compares the hazard between M3C and PRISK (Principia Mechanica Ltd, 1985) using the source model constructed for a nuclear site in southern England by the Seismic Hazard Working Party (SHWP, 1987). Here, I will compare M3C with OpenQuake (Pagani et al., 2014), a recent software package for seismic hazard assessment, that an increasing number of analysts use for seismic hazard projects (e.g. Bommer et al., 2013). Mosca et al. (2015) compare M3C and OpenQuake using a source model developed for southeastern Canada by Atkinson and Goda (2011). Although the motivations of Mosca et al. (2015) and this work are similar, in the present report we compare extensively more elements of PSHA, how they were implemented in the two software packages and whether they produce the same hazard results, using the source model developed for the UK by Musson and Sargeant (2007).

In Section 2, I describe similarities and differences between M3C and OpenQuake in terms of the IT functionality and the methodology. Section 3 describes the source zone model for the UK of Musson and Sargeant (2007). Section 4 shows the hazard results computed from the two software packages and Section 5 provides general conclusions.

2 Overview of the software packages

M3C is a computer programme developed in the BGS and routinely used for commercial and academic projects. It is based on a Monte Carlo approach for assessing the seismic hazard (e.g. Musson, 1999, 2000, Musson and Sargeant, 2007; Musson, 2009, 2012).

Once a source zone model is constructed, including the earthquake recurrence statistics for each source zone, the code generates synthetic catalogues of N-years using Monte Carlo simulations (i.e. generator of random numbers). Each simulated catalogue represents a version of what could occur based on past observed seismicity. The ground motion at a specific site is computed for each synthetic catalogue. This process is iterated R times in order to simulate millions of years of data and therefore resolve the hazard accurately for long return periods. For example, to estimate the hazard for a return period of 10,000 years, the user simulates 100,000 catalogues of 100 years, or 200,000 catalogues of 50 years, giving a total number of 10,000,000 years. To find the ground motion that has an annual probability of being exceeded by 1 in 10,000, the user sorts the values in order of decreasing severity and picks the 1001st value. This has been exceeded 1,000 times out of 10,000,000 and therefore has a 1 in 10,000 probability of being exceeded (Figure 1; Musson, 2000). Using the same procedure, it is possible to identify ground motions associated with different return periods.

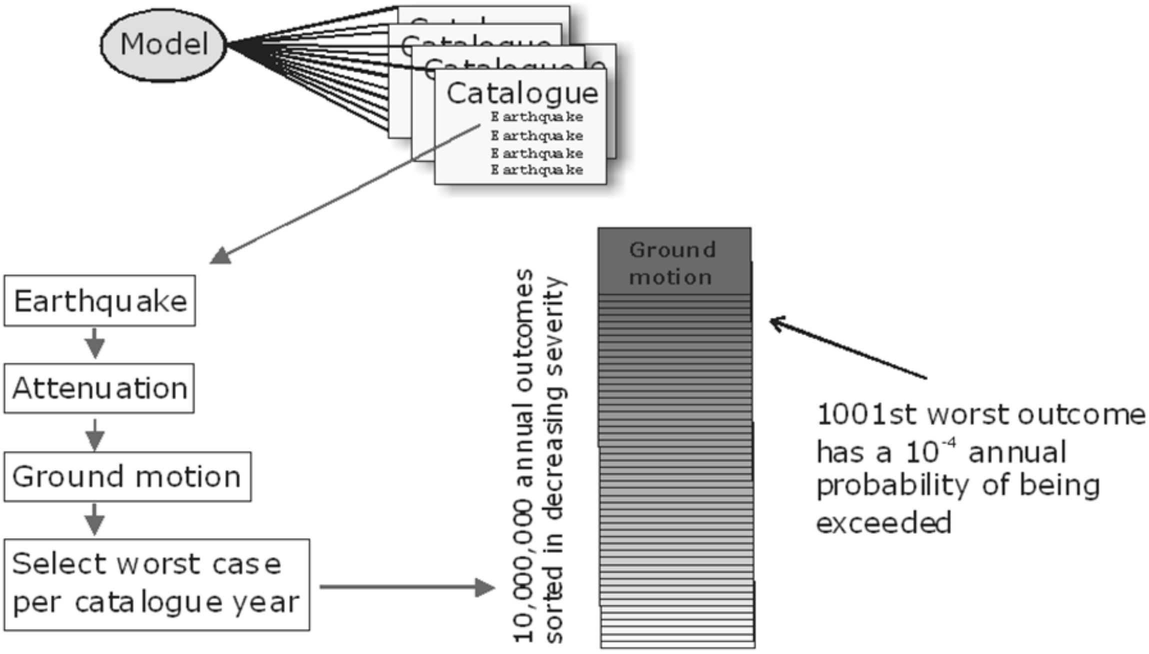

Figure 1: **The elements of the Monte Carlo simulation approach to probabilistic seismic hazard assessment (from Musson, 2000).**

The Global Earthquake Model initiative has developed OpenQuake (Crowley et al., 2013). This is an open-source software suitable for a large range of applications and allows the user to make hazard and risk calculations at various scales, from single sites to large regions. In this work, I test and analyze only the hazard module of OpenQuake. The software offers multiple types of hazard calculations: Cornell-McGuire PSHA as proposed by Field et al. (2003), a Monte Carlo based PSHA with a set of stochastic events and ground motion fields for each rupture, and deterministic seismic hazard analysis for a single earthquake scenario (Pagani et al., 2014). OpenQuake uses the seismic source model to create a list of earthquake ruptures applying the Earthquake Rupture Forecast (ERF) calculator. This is combined with the chosen GMPEs and a tectonic region to compute the hazard curves for the specific site(s) for the Cornell-McGuire PSHA (GEM, 2019). For the Monte-Carlo based PSHA, the ERF is used to generate a set of stochastic events by sampling the ruptures included in the ERF according to their probability of occurrence. Then, the set of stochastic events is associated with the chosen GMPE to have the ground motion value. The reader can refer to Pagani et al. (2014) and GEM (2019) for details.

2.1 COMPARISON OF IT FUNCTIONALITIES

Table 1 summarizes the comparison between IT functionalities of M3C and OpenQuake.

	M3C	**OpenQuake**
Version	3.14	2.8
Developers	Musson and Mosca	Pagani et al.
Code availability	Free upon request	Open-source, https://www.globalquakemodel.org/oq-getting-started
Program language	FORTRAN	Python
I/O format	ASCII	NRML
Platform	Windows, Unix, macOS	Ubuntu, Linux, macOS, and Windows
Number of processors	Single processor	As many processors as available
Documentation	User Manual	User Manual
GUI	No	No

Table 1: Comparison of the computational engine of M3C and OpenQuake.

M3C is a FORTRAN computer program, the input/output (I/O) format is ASCII, and it runs on both Windows, UNIX and macOS platform using one processor. M3C is available upon request.

OpenQuake's engine is more complex because it has many levels of modularity. The programming language is Python and the format of I/O information is a customized XML schema called Natural Hazard Risk Markup Language (see Pagani et al. (2014) and www.globalquakemodel.org/openquake/ for more details). This software is available for the Linux, macOS, and Windows platforms and uses as many processors as are available. The source code can be downloaded from a public web-based repository (http://github.com/gem/oq-engine).

Both codes have a modular and flexible structure that ensures it is possible to incorporate new features. In M3C, the modular structure consists of FORTRAN subroutines for the various steps of PSHA (e.g. GMPEs, generating synthetic catalogues). The OpenQuake engine consists of a number of self-sufficient libraries, e.g. oq-hazardlib for the hazard calculations, oq-risklib for the risk calculations, oq-nrmlib to read, write, and validate input and output files (Pagani et al., 2014).

The two software packages provide a user manual. Neither of them offers a graphic user interface and interact with the user through the command line interface. OpenQuake is associated with pre- and post-processing libraries, e.g. OQ strong motion toolkit for the basic analysis of strong motion recordings, OQ Catalogue Toolkit for homogenising different earthquake catalogues, OQ hazard Toolkit for building the source model (Weatherill et al., 2016).

2.2 COMPARISON OF THE METHODOLOGY

Table 2 summarizes the comparison between the methodological aspects of M3C and OpenQuake and the subsections below describe them extensively.

	M3C	**OpenQuake**
PSHA Approach	Monte Carlo based PSHA	Cornell-McGuire PSHA, Monte Carlo based PSHA
Gutenberg-Richter relationship	Yes	Yes
Activity rate	Computed for Mw=0 and ≠0	Computed for Mw=0
Earthquake rupture modelling	Rupture finiteness in 3-D for fault and 2-D for areal sources	Rupture finiteness in 3-D for fault and areal sources
Type of magnitude-scaling relationship	Any magnitude-length scaling relationship	Magnitude-area scaling relationship of Wells & Coppersmith (1994), Thomas et al. (2010), EPRI (2011), and Strasser et al. (2010)
GMPE implementation	Built-in	Built-in
Truncation of the GMPE variability	Yes. Option not to truncate the GMPE variability	Yes
Treatment of epistemic uncertainty	Logic tree and *pdf*	Logic tree
Outputs	Hazard curves and maps, UHS, disaggregation for M-R-ε	Hazard curves and maps, UHS, disaggregation for M-R-ε-Location

Table 2: Comparison of M3C and OpenQuake in terms of methodology. M-R-ε indicates magnitude (M), distance (R), and the number of standard deviations above or below the ground motion median prediction (ε).

2.2.1 Type of PSHA

From a methodological point of view, M3C performs a Monte Carlo based PSHA, whereas OpenQuake uses Cornell-McGuire PSHA or Monte Carlo based PSHA.

2.2.2 Seismic source

OpenQuake models sources as points, lines (faults) and areas, whereas M3C models fault and area sources but not point sources.

2.2.3 Fault Rupture

In OpenQuake, the finite-fault rupture is modelled as 3-D rectangular planes for both fault and area sources. The plane is described by the nodal plane orientation (i.e. strike, dip and rake), upper and lower depths of the seismogenic zone, rupture aspect ratio, magnitude scaling relationship, and Gutenberg-Richter recurrence law (Pagani et al., 2014; Monelli et al, 2014).

In M3C, the same parameters are required for modelling the finite-fault ruptures for fault sources, but not for areal sources where the fault rupture is modelled as a line in a 2D space. For areal sources, each synthetic epicentre is generated in an area source zone and located at the centre of a finite fault rupture. The size of the rupture is computed using the magnitude of the synthetic event, the magnitude-scaling relationship, the fault orientation (if known), and the faulting style. If the fault orientation is unknown, random orientations are considered (Musson, 2009).

2.2.4 Magnitude scaling relationship

OpenQuake uses the magnitude-area scaling relationship:

$$Mw = b * \log A + a \tag{1}$$

Where A is the area of the fault rupture, Mw is the moment magnitude, and *a* and *b* are the regression coefficients. OpenQuake supports the magnitude-area scaling relationship of Wells and Coppersmith (1994) based on a global database of earthquake ruptures, Strasser et al. (2010) for interface and in-slab earthquakes, Thomas et al. (2010) developed by the Pacific Earthquake Engineering Research (PEER) for the validation of PSHA programs, and EPRI (2011) for the central and eastern United States.

M3C is more flexible because the user can input the coefficients *a* and *b* of any magnitude-length scaling relationship:

$$Mw = b * \log L + b \tag{2}$$

Where L is rupture length. Monelli et al. (2014) find that the use of a magnitude-area scaling relationship rather than a magnitude-length scaling relationship explains differences in the hazard results. In OpenQuake, the fault rupture is created by conserving the rupture area computed using the magnitude-area scaling relations and a specific rupture magnitude. This means that the rupture length may be increased for a given aspect ratio and rupture area if the width of the fault rupture is larger than the seismogenic thickness (Monelli et al., 2014; Pagani et al., 2014). In M3C, the rupture extension is constrained by the magnitude-length scaling relations and therefore there is one rupture distance for the same rupture magnitude.

2.2.5 Faulting style and depth

Both software packages implement the predominant faulting style (i.e strike-slip, thrust and, normal) for each seismic source. This is defined by predominant faulting style and strike in M3C, and rake, dip and strike in OpenQuake. M3C and OpenQuake assign a depth distribution to each seismic source.

2.2.6 Magnitude-frequency distribution

Seismicity is modelled as a Poisson process in both codes and the magnitude-frequency distribution is described by a double truncated Gutenberg-Richter distribution, which is bounded by a minimum magnitude and a maximum magnitude. It is worth noting that the activity rate for the seismic sources is defined for 0.0 Mw in OpenQuake and a minimum magnitude that can be 0.0 or non-0.0 in M3C. Furthermore, the OpenQuake engine allows the possibility of using other magnitude-frequency distributions, such as the hybrid characteristic earthquake model of Youngs and Coppersmith (1985) and an “arbitrary” distribution.

2.2.7 Ground motion models

The implementation of the GMPEs in M3C and OpenQuake is very similar. A large number of ground motion models are built-into the software, i.e. they are implemented as stand-alone functions in their own sub-routine. Ground motion truncation is supported by both codes. In M3C it is possible to select the untruncated ground motion model. To reproduce the same condition, the truncation level should be set to 6 in OpenQuake (Pagani et al., 2014).

2.2.8 Epistemic uncertainties

Epistemic uncertainty describes the scientific uncertainty in the simplified model and reflects our lack of knowledge regarding earthquake processes. They are expressed by a logic-tree where each branch is set up for alternative models, parameters and assumptions. Weights are given to each branch to reflect the relative confidence that the analyst has in that model. OpenQuake and M3C implement the logic tree approach for epistemic uncertainties.

The treatment of epistemic uncertainties in Cornell-McGuire based PSHA and Monte Carlo based PSHA is different. In the Cornell-McGuire approach, the hazard results are performed for every possible combination of branches and the outcome represents a weighted mean (e.g. McGuire, 2004; Musson, 2012). In a Monte Carlo-based PSHA, not all possible values of the logic tree branches are computed but they are sampled randomly based on their weights and a single hazard calculation is performed (Musson, 2012). For this reason, in M3C, it is straightforward to write an input file that contains many branches in the logic tree, whereas in OpenQuake a large logic tree produces a lengthy, and often impractical, input file for both Cornell-McGuire PSHA and Monte Carlo-based PSHA.

2.2.9 Hazard outputs

In terms of seismic hazard outputs, both M3C and OpenQuake compute seismic hazard curves and maps, uniform seismic hazard spectra (UHS), and disaggregation. It is worth noting that the grid spacing for the seismic hazard maps is in degrees in M3C and kilometres in OpenQuake. For this reason, it is not straightforward to compare the hazard maps produced by the two codes for the exact number of grid points.

3 Data

In this section, I describe briefly the source zone model for the British Isles developed by Musson and Sargeant (2007) that is the basis for comparing the two software packages.

The UK source zone model was used to produce the most recent UK national hazard maps for the building code Eurocode 8 (Musson and Sargeant, 2007). The model, which consists of 23 source zones, is strongly based on the tectonics and kinematics of the UK and less influenced by the seismicity distribution (Figure 2). It also includes the Viking Graben and associated structures as a single zone but excludes some parts of Scotland, extreme north-east of England, the Isle of Man, Northern Ireland, and the offshore area around the UK due to the low seismicity level (Musson and Sargeant, 2007). The site for performing most hazard calculations has been chosen arbitrarily and is situated in the city of Manchester, i.e. 52.48°N and -1.89°E. However, I will also discuss the hazard curves determined for various other sites in the UK (Figure 2).

The Gutenberg-Richter recurrence law for the UK source model was computed using the penalised maximum likelihood in Johnston et al. (1994) and modified by Musson (2011). It maximizes the information provided by different time windows of the earthquake catalogue for different magnitude completeness thresholds and allows a prior value to constrain the *b*-value (i.e the proportion of large events to small ones) in zones where there are few earthquakes. Using this method, Musson and Sargeant (2007) compute the recurrence statistics for the individual source zones of the UK model. The activity rate *a* (a function of the total number of earthquakes in the sample) and the *b*-value of each source zone are expressed by a *pdf* and discretized by 25 pairs of the recurrence parameters with associated weights. Figure 3 shows an example of the probability distribution for the source zone EC9M and Table 3 shows the most likely value of the recurrence parameters in the distribution of the source zones.

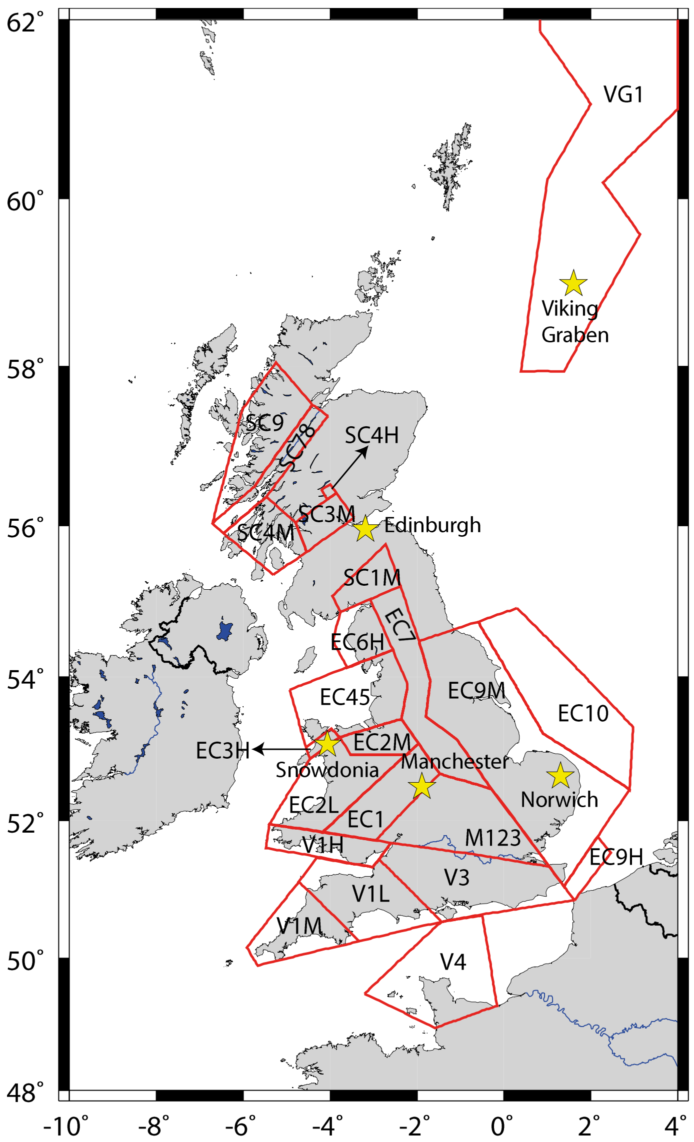

Figure 2: Source zone model of the UK from Musson and Sargeant (2007). It consists of 23 zones and the yellow stars indicate various sites.

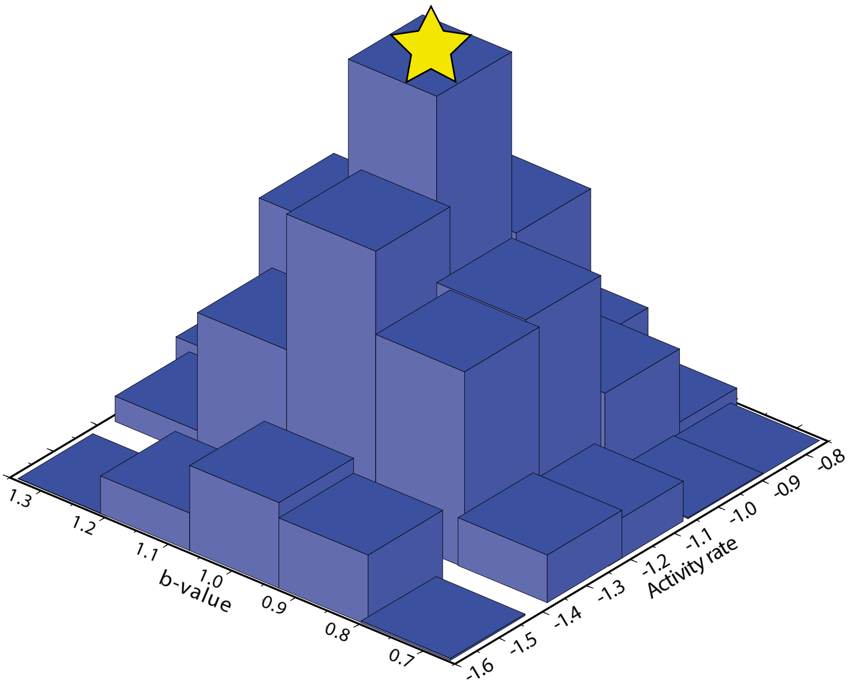

Figure 3: Probability density function for the activity rate and the *b*-value of the source zone EC9M of the UK source model. The star indicates the most likely value for the recurrence parameters (a = -1.15 and *b* = 1.00).

	Activity rate	*b*-value		Activity rate	*b*-value
SC1M	-1.93	1.00	EC7	-0.71	0.90
SC3M	-1.11	1.00	EC9H	-1.41	0.85
SC4H	-1.37	0.96	EC9M	-1.15	1.00
SC4M	-1.02	1.03	EC10	-0.82	1.01
SC78	-0.84	1.03	M123	-1.90	1.00
SC9	-0.83	1.09	V1H	-1.15	0.81
EC1	-0.92	0.92	V1M	-0.98	1.06
EC2M	-1.43	1.06	V1L	-1.11	1.00
EC2L	-1.62	1.00	V3	-1.64	0.98
EC3H	-1.36	0.85	V4	-1.01	0.77
EC45	-1.09	1.01	VG1	0.07	1.07
EC6H	-1.65	0.97			

Table 3: Activity rate with respect to 3.0 Mw and *b*-value for the 23 source zones of the UK model. The recurrence parameters were estimated using the penalized maximum likelihood procedure of Johnston et al. (1994).

Although the user can include a large number of recurrence parameters in the input file for OpenQuake and therefore construct a logic tree with many branches, it will end up in a lengthy file and a huge logic tree, consisting of 25 recurrence parameters multiplied by the 23 source zones multiplied by the branches from other parameters (e.g. GMPEs and maximum magnitude).

Maximum magnitude (Mmax) in the UK source model is defined by two logic trees, depending on whether the source zone is offshore or onshore (Table 4). The minimum magnitude that is the magnitude of the smallest earthquakes considered to be of engineering significance is chosen to be 4.5 Mw. Table 5 shows the depth distribution. For all source zones, the faulting is associated with a strike-slip focal mechanism with equal probability of having either a north-south or east-west orientation (Musson and Sargeant, 2007).

To check whether the implementation of the GMPEs in M3C and OpenQuake provides similar hazard results in spite of the differences in the engine, I test many ground motion models, each associated with a weight of 1.0, as well as a combination of them in a weighted logic tree. I have chosen GMPEs that are commonly used for seismic hazard studies in the UK and worldwide, including Akkar et al. (2013), Boore et al. (2014), Abrahamson et al. (2014), and Chiou and Youngs (2014). However, in most tests described in the next section, I use the ground motion model of Boore et al. (2014) that is from the “Next Generation Attenuation 2” project conducted by PEER in the western United States (Bozorgnia et al., 2014). I applied a ground motion truncation of 3σ to the hazard calculations.

The site condition is assumed to be class B of the NEHRP (1994) classification. I, therefore, assign a V_{S30} value (i.e. average shear wave velocity in the top 30 m) of 760 m/s.

Mmax	**Weight (onshore)**	**Weight (offshore)**
5.5	0.20	-
6.0	0.50	0.60
6.5	0.30	0.40

Table 4: Distribution of the maximum magnitude of the UK model, together with their weight.

Depth [km]	**Weight**
5	0.10
10	0.25
15	0.40
20	0.25

Table 5: Distribution of the focal depth of the UK model, together with their weight.

4 Results

This section presents the comparison of the hazard calculations for the PEER validation exercises of Thomas et al. (2010) and for the UK source model.

4.1 PEER VALIDATION TESTS

I used the exercises developed within the PEER Centre’s Lifelines Program (Thomas et al., 2010) as a first step to validate the computer program M3C. This set of exercises is designed to check how the codes implement fundamental steps of PSHA, e.g. implementation of the magnitude-frequency distribution, modelling of the area sources and ruptures on the fault planes. For this reason, they use a single source typology and no epistemic uncertainty. Below, I show the results for Set 1 Case 5 (referred as to S1C05) and Set 1 Case 10 (referred as to S1C10). In this comparison, I did not consider OpenQuake because Pagani et al. (2014) show the results of the PEER validation exercises using OpenQuake.

S1C05 tests a vertical strike-slip fault with uniform slip, b-value of 0.9 and an activity rate of 3.129 for a minimum magnitude of 0.0 Mw, and the truncated exponential magnitude distribution between 5.0 and 6.5 Mw. Figure 4a shows the configuration of the seismic fault source and four sites. The magnitude-length scaling relationship is $log L = 0.5 Mw – 1.85$. The length of the fault is 25.0 km and the width of the fault plane is 12.0 km. This test assumes that the fault rupture is smaller than the entire fault length. The GMPE used for the hazard calculations is Sadigh et al. (1997) for rock soil conditions and the ground motion truncation is 0σ. I simulated 1,000,000 earthquake catalogues, each 100 years long. The total number of 100,000,000 years is sufficient to resolve the hazard accurately for the annual probability of exceedance (APoE) of 10^{-6}. Figure 4b-4e shows the PGA hazard curves computed by M3C and the comparison with the solutions in Appendix A of Thomas et al. (2010) that are the mean values of the distribution estimates from the software packages considered in Thomas et al. (2010). The comparison is very good when the site is outside the fault, whereas there are some discrepancies between the hazard results from M3C and the solution of Thomas et al. (2010) when the site is located along the fault. It is worth underlining that this is a rare case and a site next to the fault, but not along the fault, is more common.

S1C10 considers a uniform area source with a truncated exponential magnitude distribution between 5.0 and 6.5 Mw, b-value of 0.9 and activity rate of -1.403 for a minimum magnitude of 5.0 Mw, and fixed the focal depth of 5 km. The sites are situated in four locations (Figure 5a). As in Test S1C05, the GMPE for this exercise is Sadigh et al. (1997) for rock soil conditions and the ground motion truncation is 0σ. I simulated the same number of catalogues as for S1C5, 1,000,000 catalogues, each 100 years long. Figures 5b-5e shows an excellent agreement between M3C and the hazard curves in Appendix A of Thomas et al. (2010). The solutions in Thomas et al. (2010) are the mean values of the distribution estimates from the software packages considered.

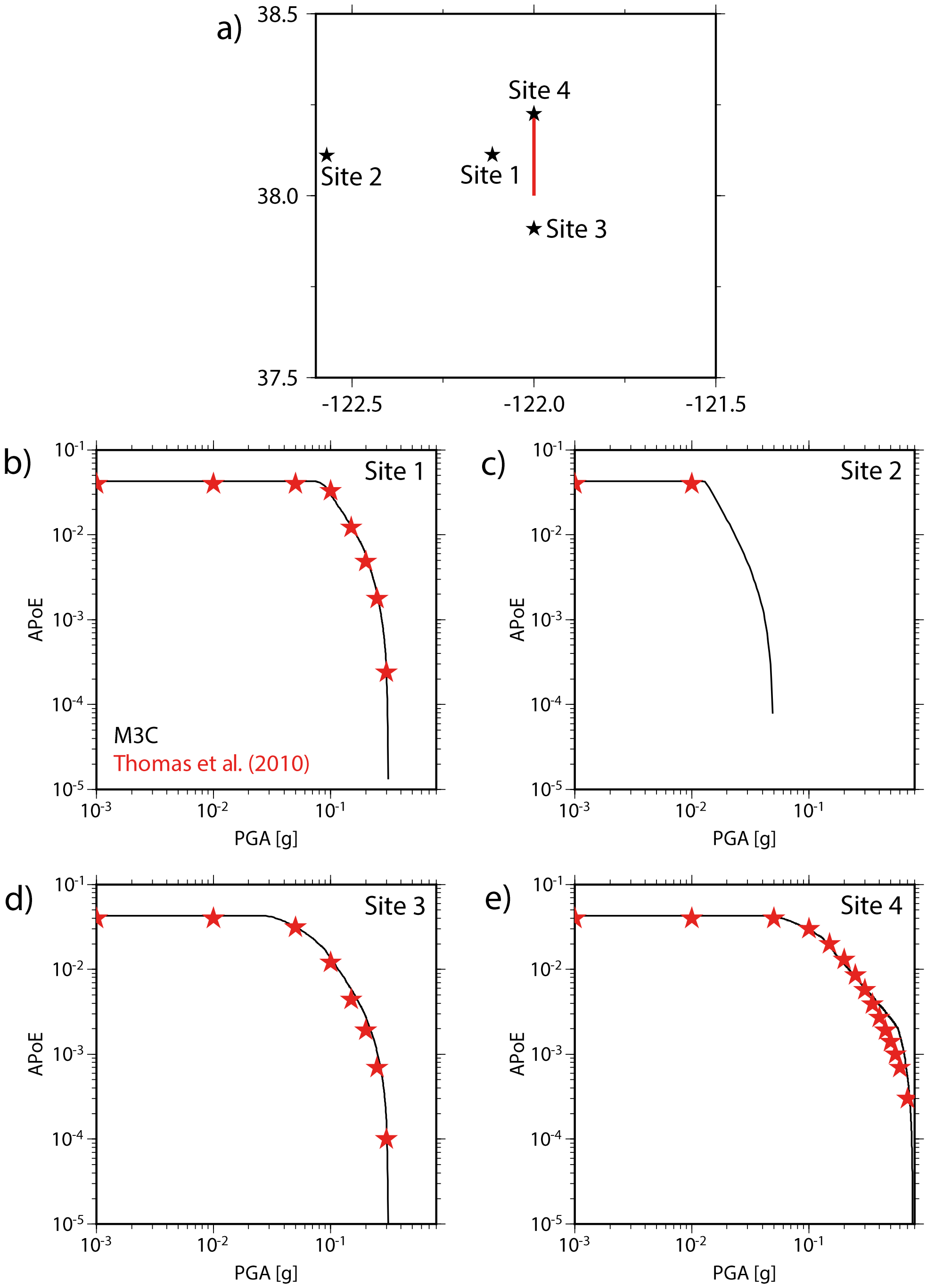

Figure 4: a) Source-to-site configuration of the validation test for the fault source in S1C05 of Thomas et al. (2010); and b-e) Comparison of the hazard curves, expressed as annual probability of exceedance (APoE), for PGA computed by M3C (black solid lines) and the PGA hazard curves in Thomas et al. (2010) (red stars).

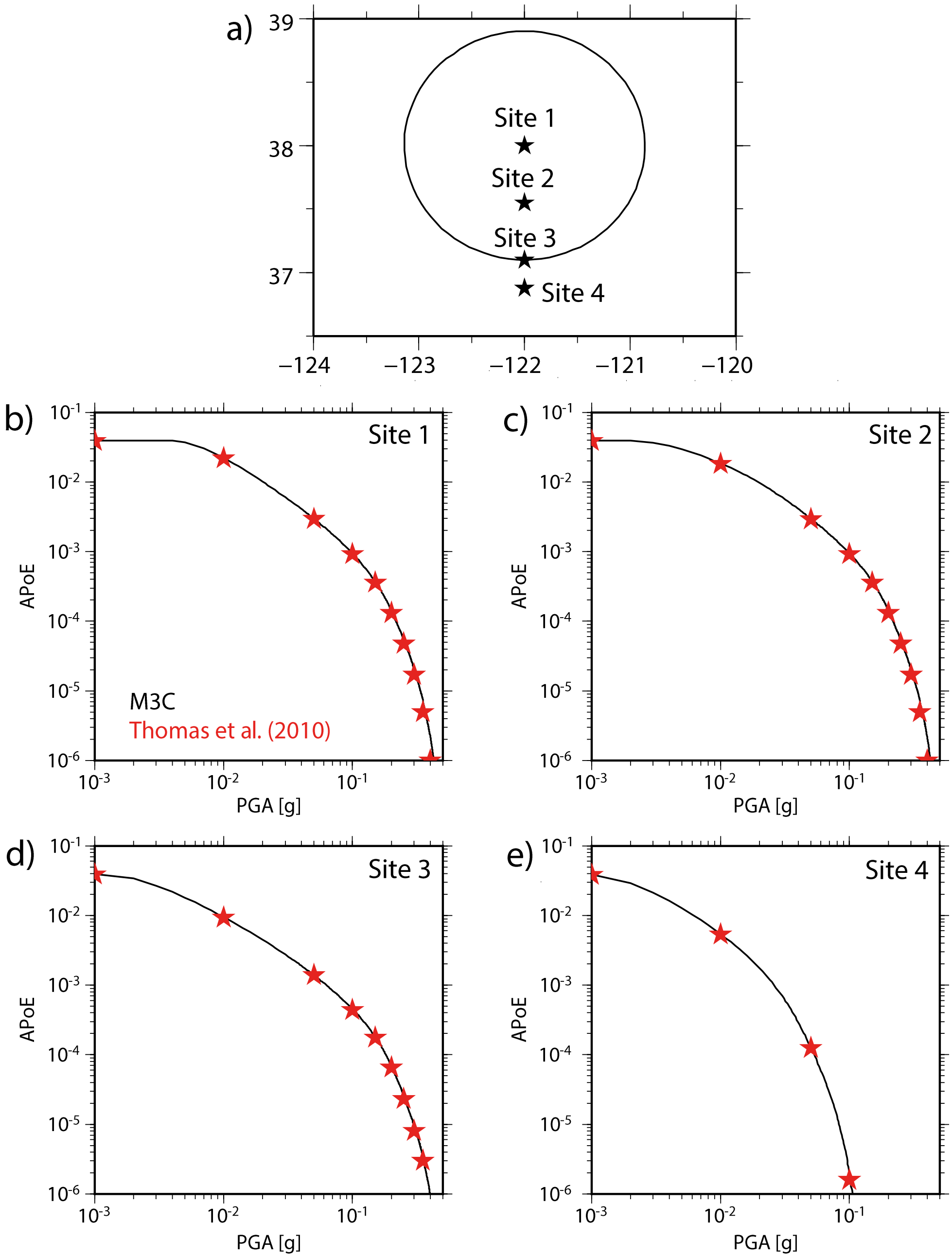

Figure 5: a) Source-to-site configuration of the validation test for the areal source in S1C10 of Thomas et al. (2010); b-e) comparison of the hazard curves for PGA computed by M3C (black solid lines) and the PGA hazard curves in Thomas et al. (2010) (red stars).

4.2 COMPARING RESULTS USING THE UK NATIONAL SEISMIC HAZARD MODEL

To test a real source model with a complex source-site configuration and finite rupture modelling, I apply the UK source model described in Section 3. I compare hazard results produced by OpenQuake and M3C.

I used M3C to generate 1,000,000 synthetic catalogues each 100 years for the source zone model of the UK. 100,000,000 years of data is enough to resolve long return periods, up to 10,000 years. In the following set of tests, I used OpenQuake to implement the Cornell-McGuire PSHA and therefore it did not simulate synthetic catalogues. To model the fault rupture for M3C, I used the magnitude-length scaling relationship of Wells and Coppersmith (1994) for strike-slip faulting and "Subsurface rupture length" Mw=*1.49 logL + 4.33* (referred here as to WC94/SSRL) and an aspect ratio equal to 1.0. OpenQuake uses the magnitude-area scaling relationship of Wells and Coppersmith (1994) for strike-slip faulting, i.e. *log A = -3.42 + 0.90*

Mw, and the aspect ratio of 1.0. In all tests below, except Test 1, I used the GMPE of Boore et al. (2014).

M3C takes around 5 minutes to generate 1,000,000 synthetic catalogues and compute the hazard for a site, using a single processor, whereas OpenQuake takes about 7.4 minutes to perform the Cornell-McGuire PSHA using eight processors. Although the computational time of M3C and OpenQuake to make a hazard calculation is similar (5.0 versus 7.4 min), M3C uses only one processor, whereas OpenQuake uses eight processors. The performance of OpenQuake improves significantly as the number of available processors increases.

To make a quantitative evaluation of the difference between the results, I made a number of tests and estimated the relative difference Δ between pairs of annual probability of exceedance with the same ground motion parameter:

$$\Delta = (\text{APoE2} - \text{APoE1})/\text{APoE1} \tag{3}$$

Where APoE1 and APoE2 are the annual probability of exceedance from M3C and OpenQuake, respectively. This function varies between -1.0 and 1.0. When $\Delta=0.0$, the hazard curves are identical and as the absolute values of Δ increases, the difference between the hazard curves increases. It is difficult to assess what an acceptable difference is when comparing results from different codes. However, McGuire (2012) and USNRC (2012) suggest that for a site-specific PSHA, a change in APoE of less than ±25 % may not be significant when $\text{APoE}<10^{-4}$, and the tolerance increases to ±35 % for $\text{APoE}>10^{-6}$ (Bommer et al., 2013).

4.2.1 Test 1: GMPEs

In the first test, I tested various GMPEs to check whether the implementation of the ground motion models in M3C and OpenQuake provides identical results. The GMPEs selected for this test were Akkar et al. (2013), Boore et al. (2014), Abrahamson et al. (2014), and Chiou and Youngs (2014). The hazard curves for the first test are plotted in Figure 6 for peak ground acceleration (PGA) and spectral acceleration (SA) at a period of 0.2 s and 1.0 s, examples of a short and long period acceleration, respectively. The hazard curves computed by M3C consist of 1000 points and the curves calculated by OpenQuake consist of 80 points for the same range of the ground motion parameter. This means that the spacing used by the codes to compute the hazard curves is different. As a result, the trend of Δ is irregular because I used only the common points. The agreement between M3C and OpenQuake is very good and the Δ values are between -0.3 and 0.1 for PGA, between -0.1 and 0.1 for 0.2 s SA and between 0.3 and 0.1 for 1.0 s SA (Figure 6). This means that the difference between the curves is between -30% and 10% for $\text{APoE} \leq 10^{-5}$ and therefore it is not significant based on the acceptable tolerance of McGuire (2012) and USNRC (2012). The GMPEs that result in the largest difference between the software packages are Chiou and Youngs (2014) and Campbell and Bozognia (2014; not shown in Figure 6) because they both use the rupture distance (R_{rup}), rather than the Joyner-Boore distance (R_{jb}). Bommer et al. (2013) find that GMPE models based on R_{rup} are more sensitive to the fault rupture modelling within areal sources than GMPEs based on R_{jb}. This conclusion is in agreement with the findings of the present report where differences in the hazard calculations between the software packages increase when GMPEs based on the R_{rup}, rather than on R_{jb}, are selected.

4.2.2 Test 2: Magnitude scaling relationship

The second test evaluates the influence of the magnitude scaling relationship on the hazard curves if I use a magnitude-length scaling relationship different from WC94/SSRL for M3C. I tested the magnitude-length scaling relationship of both Wells and Coppersmith (1994) for “Surface rupture length” (referred here as to WC94/SRL) and Leonard (2010) (referred here as to LEO10). Figure 7 shows clearly that WC94/SSRL and LEO10 provide the most similar results with the hazard curves computed using OpenQuake. The corresponding Δ values are very small for $\text{APoE} > 10^{-5}$. The hazard curves from WC94/SRL are slightly different from those computed by OpenQuake.

4.2.3 Test 3: Treatment of epistemic uncertainty in the recurrence statistics

In the third test, I checked how much the treatment of the epistemic uncertainty in the recurrence parameters influences the hazard curves. I ran M3C using the source model where the recurrence parameters of the source zones are given by the full *pdf*, i.e. 25 values for *a* and *b*; whereas, I run OpenQuake using the source model where the recurrence parameters of the zones are given by the most likely values in the *pdf* (see Section 3). The results are shown in Figure 8. The differences (up to 30%) between the hazard curves, especially for 1.0 s SA, are explained by the fact that the source models are not identical. For this reason, I tested the full *pdf* for the recurrence parameters of the source models using the two codes. To avoid a lengthy input file for OpenQuake and a long computational time, I restricted this test to the source zones EC1 and M123 that are adjacent to the site. In this case, the hazard curves are almost identical and the relative difference is between 0 and -15% (Figure 9).

4.2.4 Test 4: The effect of the site

In the fourth test, I used various sites (see Figure 2). The comparison between the hazard curves computed by M3C and OpenQuake is good as shown by the trend of Δ that is between -0.10 and 0.10, i.e. |Δ| < 10 % (Figure 10). The discrepancy between the hazard curves is slightly higher at the site in Snowdonia (Wales). This difference is because the site is included in a source zone that is too small to be properly resolved by OpenQuake.

4.2.5 Test 5: Monte Carlo-based PSHA

In the last test, I run OpenQuake for Monte Carlo-based PSHA and therefore generating a set of 1,000,000 stochastic events for the source model consisting of the zones EC1 and M123 only. This is because this approach is computationally very intensive and is not recommended for investigating large regions (GEM, 2019). Indeed, it took around two days to generate 1,000,000 stochastic events, and four days to generate 10,000,000 stochastic events, and compute the corresponding hazard curves using eight processors. The hazard curves for M3C and OpenQuake are relatively similar. At large (> 0.01) APoE, the differences between the two curves are significant, up to Δ=0.80. I have not investigated the reason for this large discrepancy due to the long computational time required to OpenQuake to run a large number of stochastic events using only eight processors.

4.2.6 Hazard maps

Figure 12 shows the hazard map for PGA with a return period of 475 years (i.e. 10% exceedance probability in 50 years) in the UK using M3C and OpenQuake. The grid spacing of the maps is 0.25° for M3C and 20.0 km for OpenQuake, covering the area between 49° and 61°N latitude and -8° and 4°E longitude. I used the GMPE of Boore et al (2014). There is a satisfactory agreement between the two maps, at least from a visual inspection, because the main features of seismic hazard in the UK are displayed in both maps (e.g. high PGA in the Viking Graben and in the region of Snowdonia). The computation of the relative difference between the maps is not straightforward because of the different size of the grid points. I used only the grid points with latitude and longitude such that:

$$|\ \text{Latitude (M3C)} - \text{Latitude (OQ)}\ | \leq 0.13^{o},$$

$$|\ \text{Longitude (M3C)} - \text{Longitude (OQ)}\ | \leq 0.13^{o}.$$

When I quantify the relative difference between the maps, the larger, absolute values of Δ (up to ± 0.5) correspond to the regions with low levels of seismicity that were excluded by the source model.

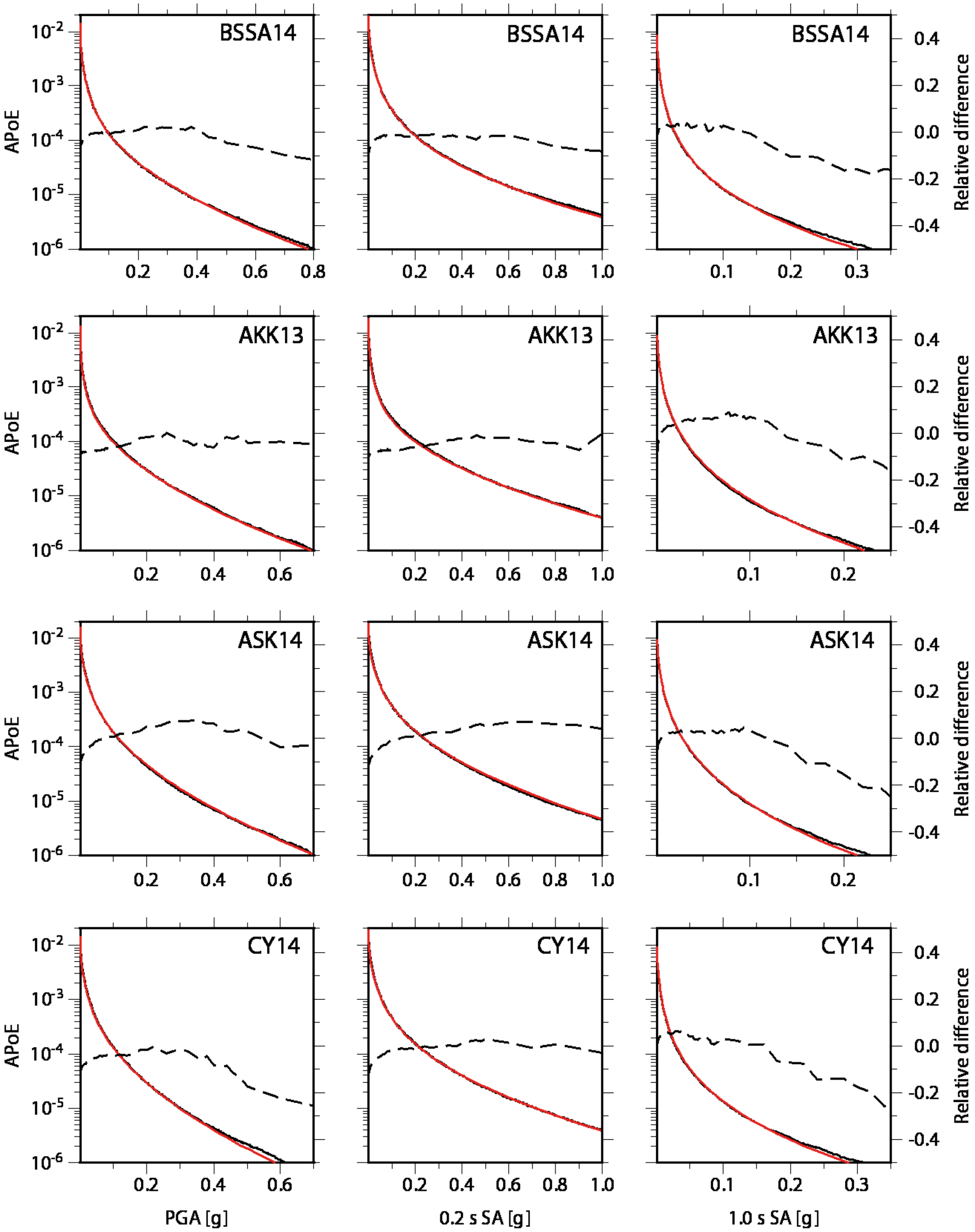

Figure 6: Hazard curves for PGA (first column), 0.2 s SA (second column) and 1.0 s SA (third column) for the source model of the UK computed using M3C (black solid lines) and OpenQuake (red solid lines), together with their relative difference from Equation 3 (dashed lines). The hazard calculations were performed for various GMPEs: BSSA14 (Boore et al., 2014), AKK13 (Akkar et al., 2013), ASK14 (Abrahamson et al., 2014), and CY14 (Chiou and Youngs, 2014).

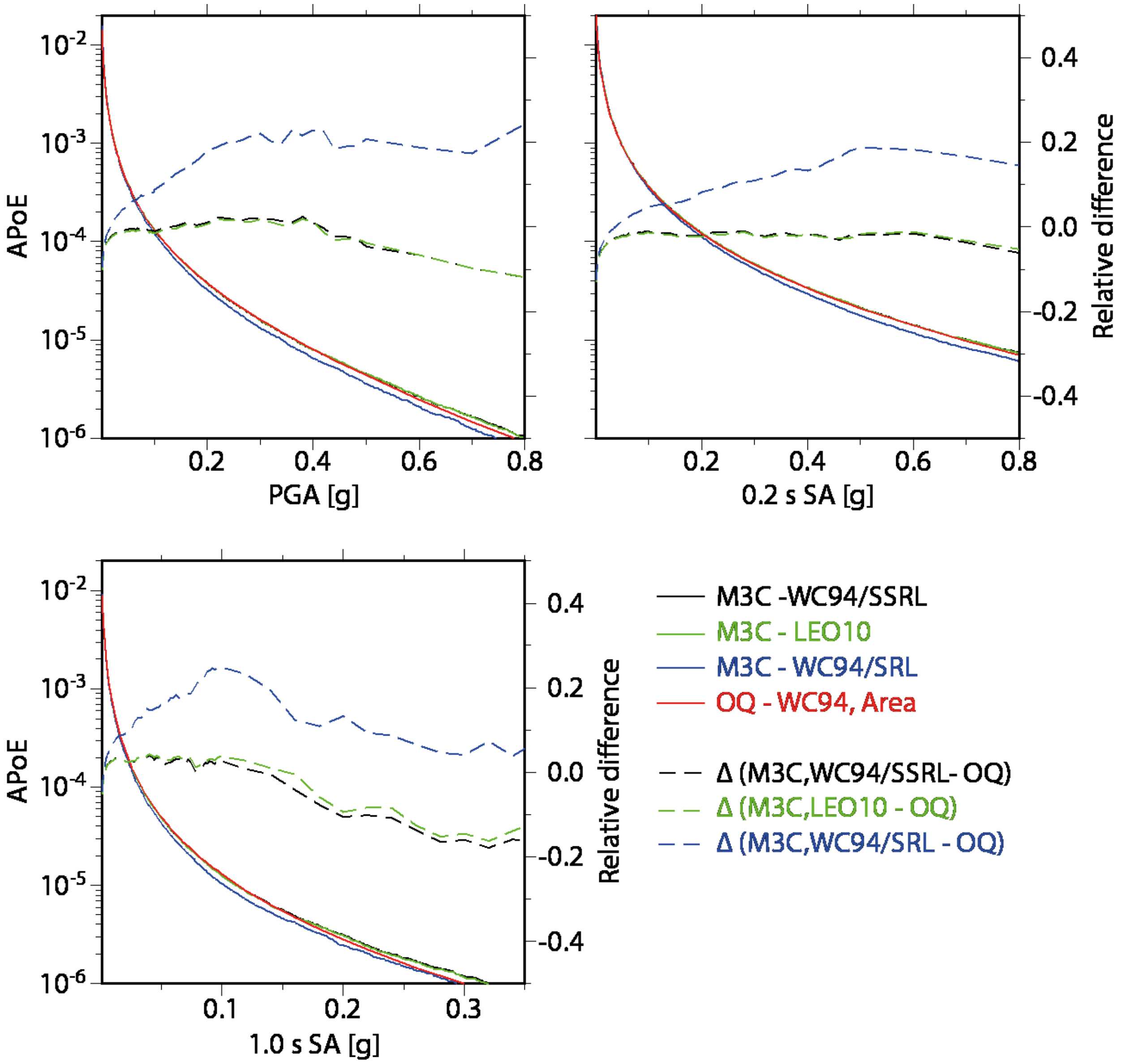

Figure 7: Hazard curves (solid lines) for PGA, 0.2 s SA and 1.0 s SA for the source model of the UK at the site, together with the corresponding relative difference (dashed lines). The black, blue, and green lines are the hazard curve computed using M3C, using the magnitude-length scaling relationship of Wells and Coppersmith (1994) for "Subsurface rupture length" (indicated as WC94, SSRL), "Surface rupture length" (indicated as WC94, SRL) and Leonard (2010) (indicated as LEO10), respectively. The red hazard curves were computed by OpenQuake, using the magnitude-area scaling relationship of Wells and Coppersmith (1994).

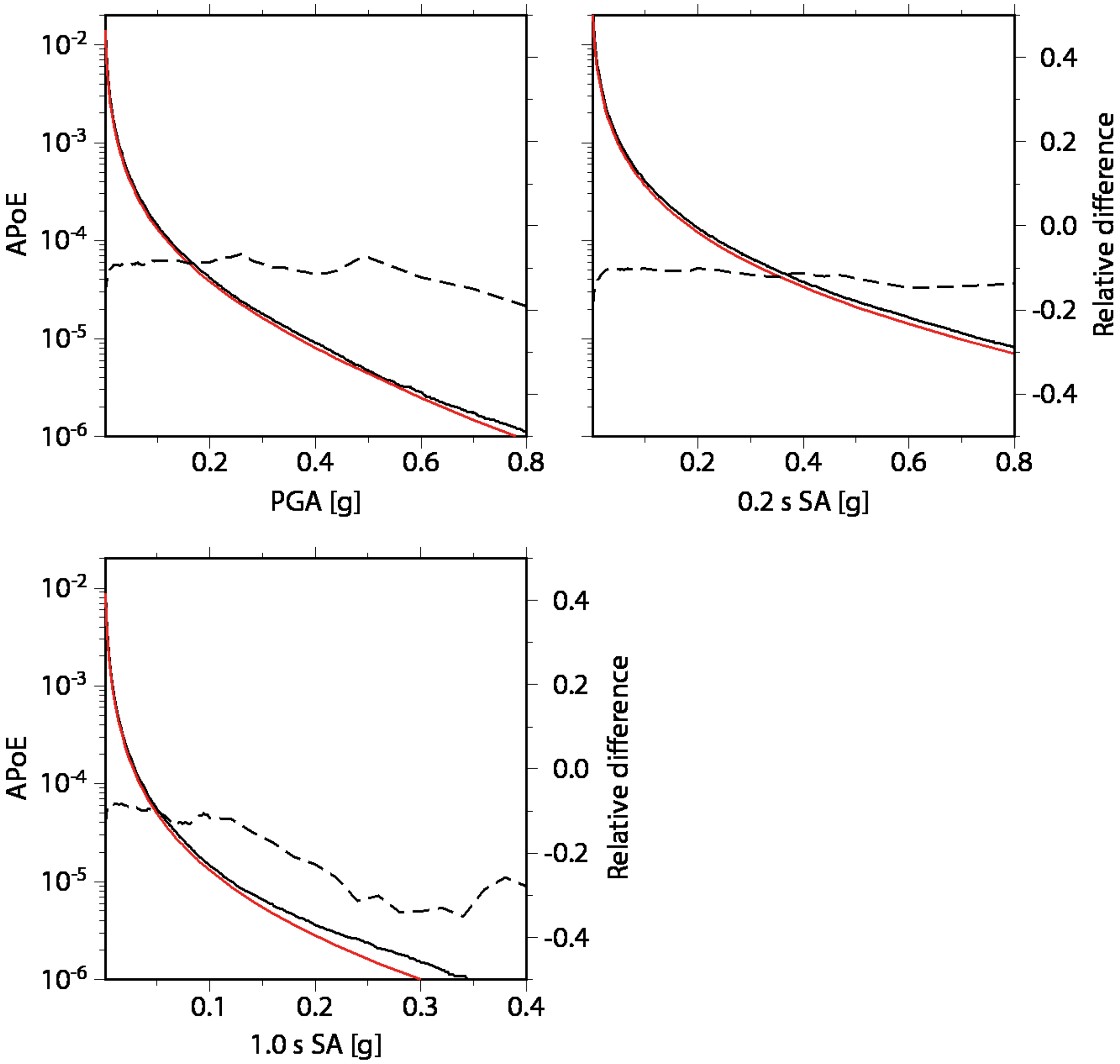

Figure 8: Hazard curves for PGA, 0.2 s SA and 1.0 s SA for the source model of the UK at the site using M3C (black lines) and OpenQuake (red lines), together with their relative difference (dashed lines). The activity rate and the b-value in the source zone model used by M3C are given by a pdf for each zone. The recurrence parameters in the source zone model used by OpenQuake are given by one value that is the most likely value in the *pdf*.

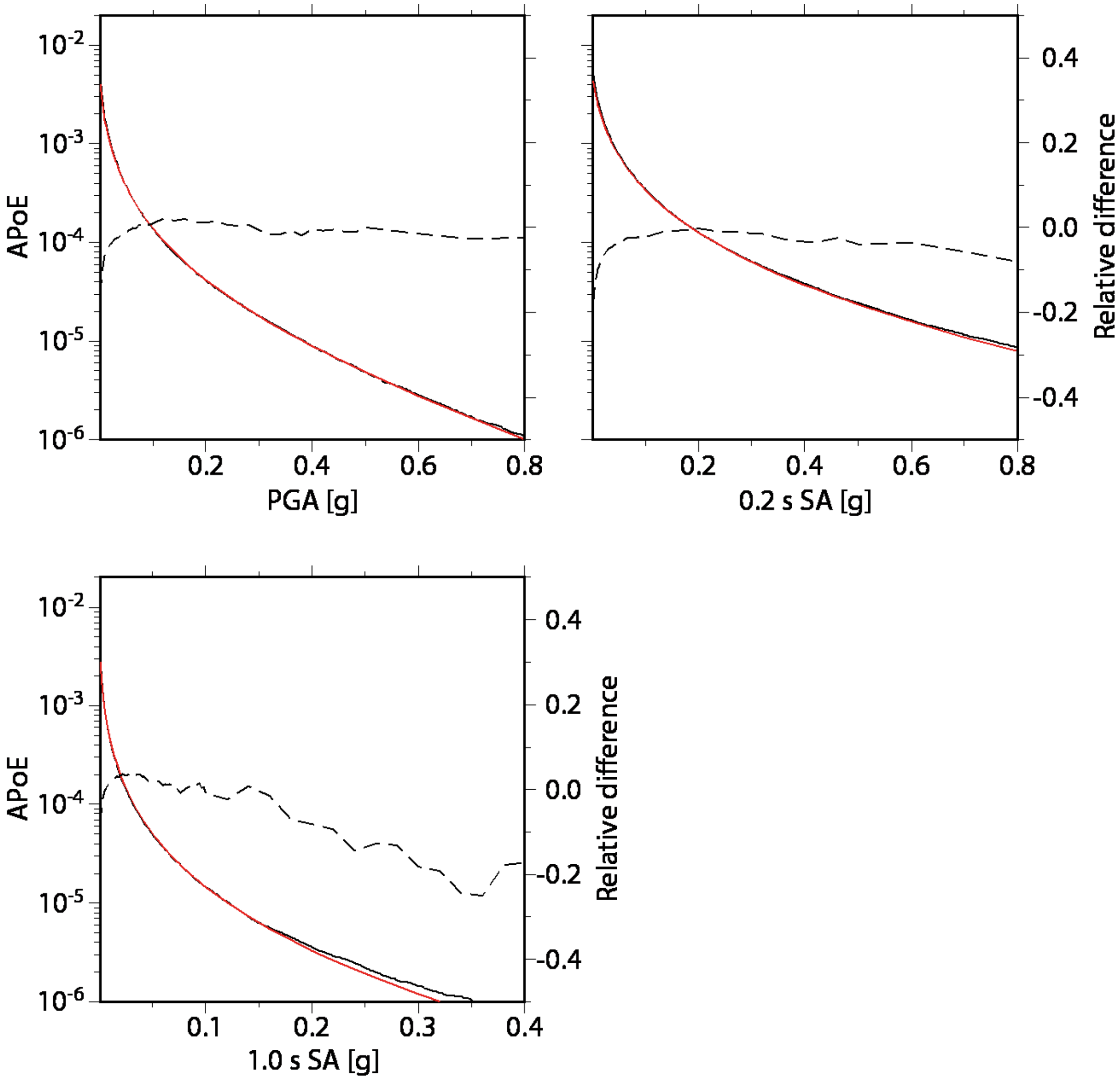

Figure 9: Hazard curves for PGA, 0.2 s SA and 1.0 s SA for two source zones of the UK source model at the site using M3C (black lines) and OpenQuake (red lines), together with their relative difference (dashed lines). The source model consists of two source zones and their activity rates and the b-values are given by the *pdf* for each zone.

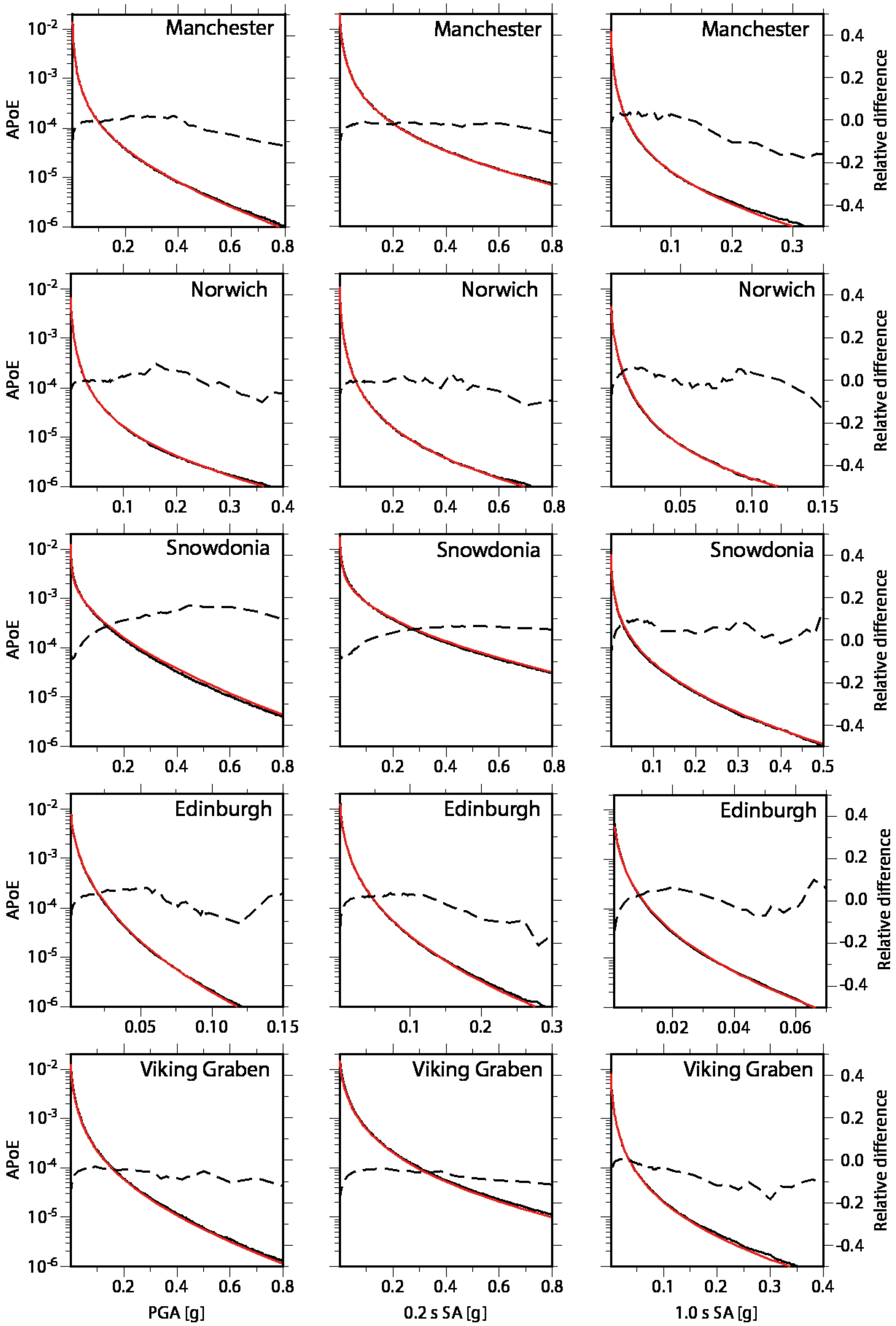

Figure 10: Hazard curves for PGA (first column), 0.2 s SA (second column) and 1.0 s SA (third column) for the source model of the UK computed using M3C (black solid lines) and OpenQuake (red solid lines), together with their relative difference from Equation 3 (dashed lines). The hazard calculations were performed for various sites (see Figure 2).

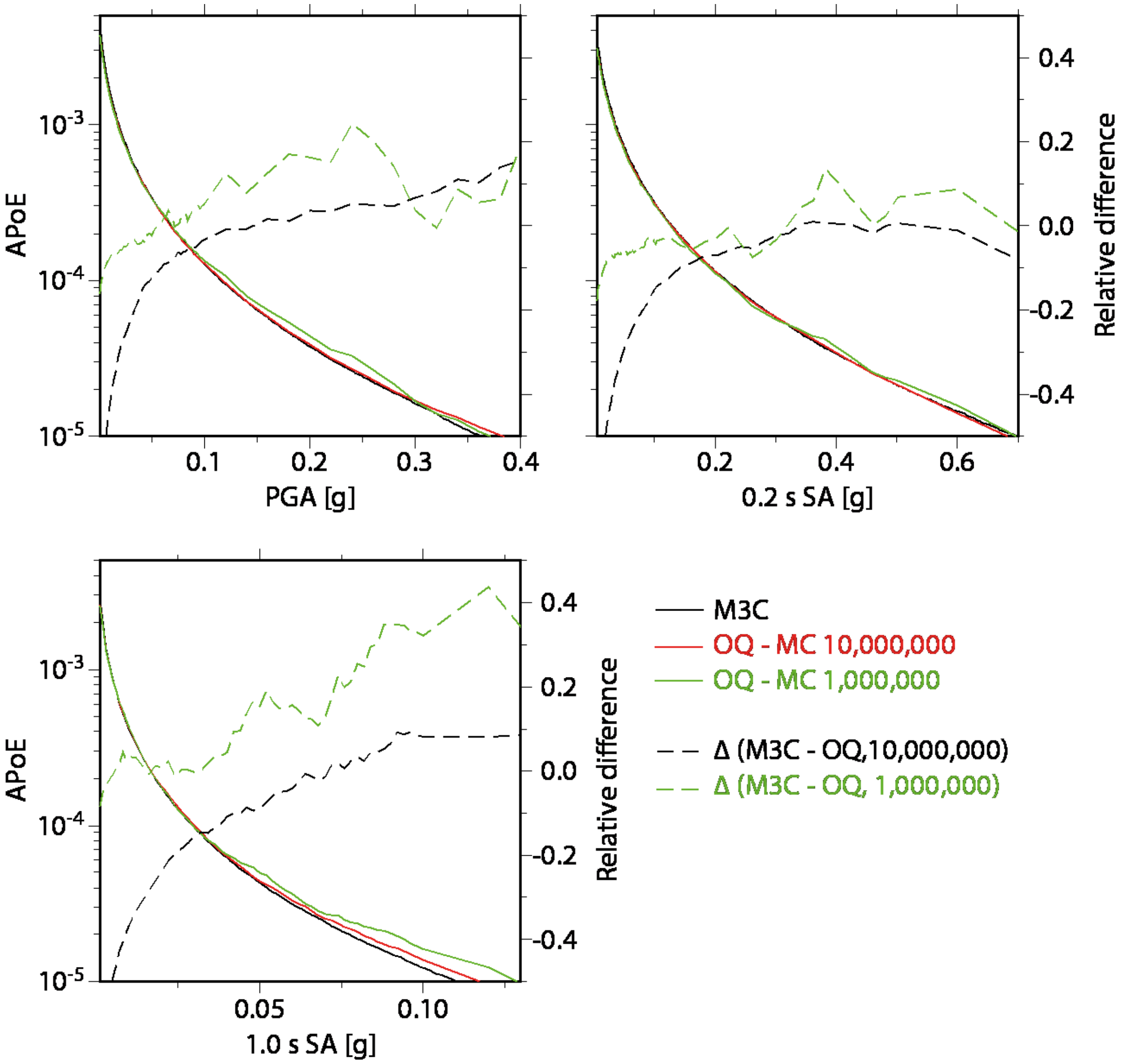

Figure 11: Hazard curves (solid lines) for PGA, 0.2 s SA and 1.0 s SA for the source model of the UK at the site, together with the corresponding relative difference (dashed lines). The black lines are the hazard curve computed by M3C. The red and green hazard curves were computed by OpenQuake, using 1,000,000 and 10,000,000 stochastic events, respectively. The source model consists of two source zone.

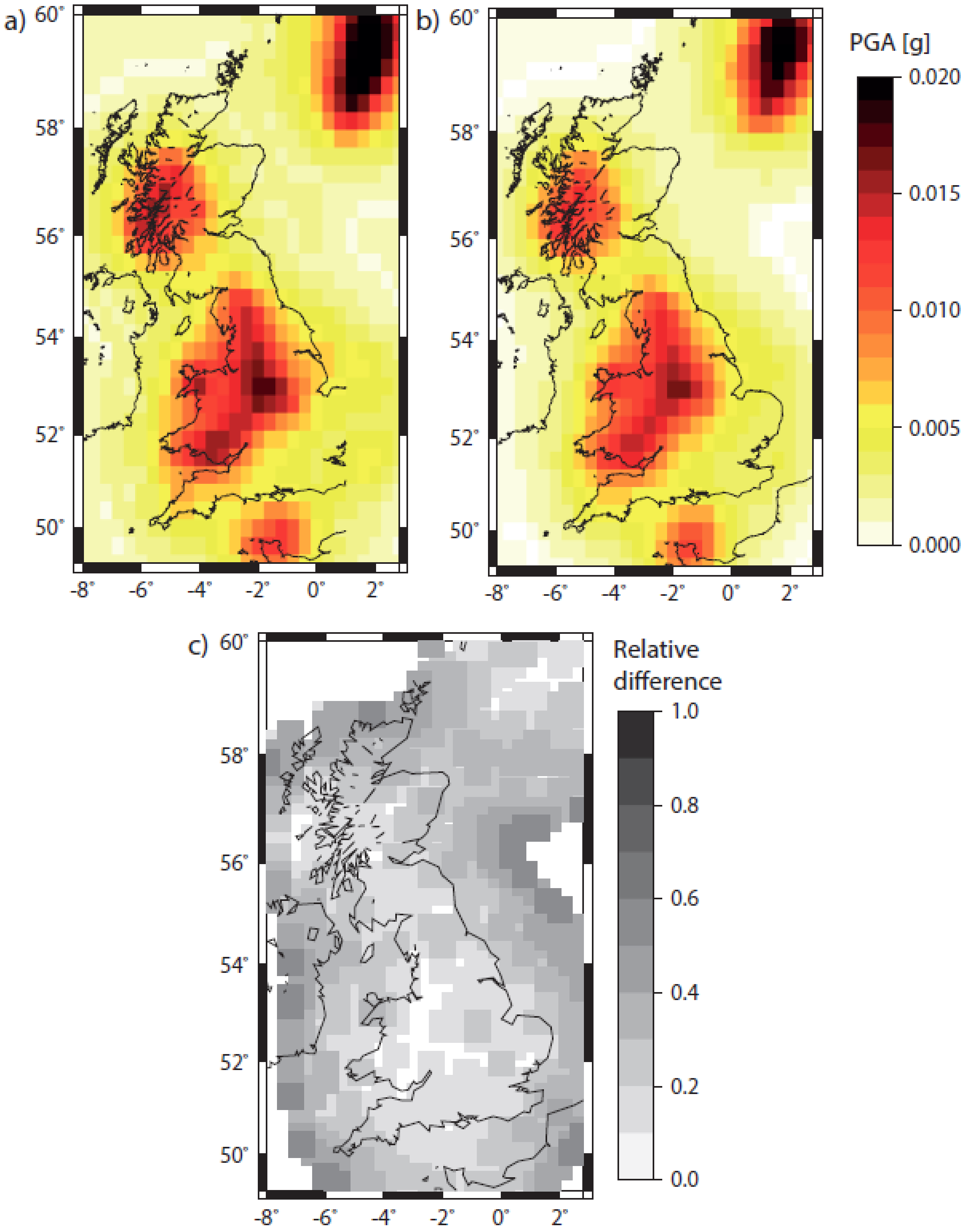

Figure 12: a) PGA hazard maps for a return period of 475 years in the UK computed using M3C; b PGA hazard maps for a return period of 475 years in the UK computed using OpenQuake; c) Relative difference between pairs of maps at the bottom. The GMPE used by the two software packages is the model of Boore et al. (2014).

5 Conclusions

The present work aimed to compare one of the most recent software packages for PSHA (OpenQuake) with the approach used in the British Geological Survey and encoded in the FORTRAN program M3C. I analyzed the methodology and the IT functionalities of the two codes (Section 2) and, then run the codes to compare the hazard for the source zone model developed for the UK (Section 3).

I tested the software packages for 1) the most common GMPE models; 2) magnitude scaling relationships; 3) the treatment of the epistemic uncertainties in the recurrence parameters. In most of the tests, M3C and OpenQuake produce similar results from a visual inspection. When I made a quantitative assessment of their difference, I found that their relative difference Δ is between -0.15 and 0.15 for an annual probability of exceedance higher than 10^{-5} that represents the range of interest for the earthquake engineering (McGuire, 2004). A range of Δ between -0.15 and 0.15 corresponds to a good tolerance level. Discrepancies between the hazard results computed by M3C and OpenQuake are explained by two factors: the different scaling relationship used in the two codes; and the use of GMPEs based on the rupture distance, rather than the Joyner-Boore distance. The fault rupture modelling is sensitive to these two factors.

Based on the results found in the present work, I conclude that the results produced by M3C and OpenQuake are in good agreement. The choice between them depends on: 1) the level of seismicity of the study area; and 2) the number of available processors for hazard calculations. In case of a region with high seismicity, the calculations performed by M3C may become computationally expensive because of a large number of simulated earthquakes for each source zone. OpenQuake becomes efficient and worth using as the number of processors increases.

Future updates of M3C should be in the following directions: implement the 3-D modelling of fault rupture also for areal sources, and develop a version of M3C that runs on several processors in order to efficiently use this software also in high seismic regions.

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